CMSC201
Computer Science I for Majors

Lecture 18 – Recursion

Prof. Katherine Gibson
Prof. Jeremy Dixon

Last Class We Covered

- Tuples
- Dictionaries
  - Creating
  - Accessing
  - Manipulating
- Dictionaries vs Lists

Any Questions from Last Time?

Today's Objectives

- To introduce recursion
- To begin to learn how to "think" recursively
- To better understand the concept of stacks

Introduction to Recursion

What is Recursion?

- In computer science, recursion is a way of thinking about and solving problems
- It’s actually one of the central ideas of CS

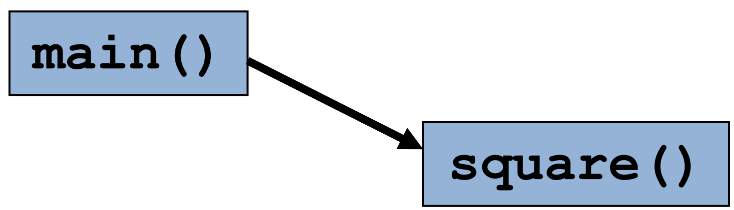
- In recursion, the solution depends on solutions to smaller instances of the same problem

Recursive Solutions

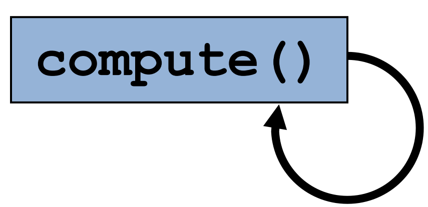
- When creating a recursive solution, there are a few things we want to keep in mind:
  - We need to break the problem into smaller pieces of itself
  - We need to define a “base case” to stop at
  - The smaller problems we break down into need to eventually reach the base case

Normal vs Recursive Functions

- So far, we've had functions call other functions
  - For example, **`main()`** calls the **`square()`** function



- A recursive function, however, calls itself



Why Would We Use Recursion?

- In computer science, some problems are more easily solved by using recursive methods
- For example:
  - Traversing through a directory or file system
  - Traversing through a tree of search results
  - Some sorting algorithms recursively sort data
- For today, we will focus on the basic structure of using recursive methods

Simple Recursion Example

```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

This is where the recursion occurs.

You can see that the `compute()` function calls itself.

This program simply computes from 50 down to 2.

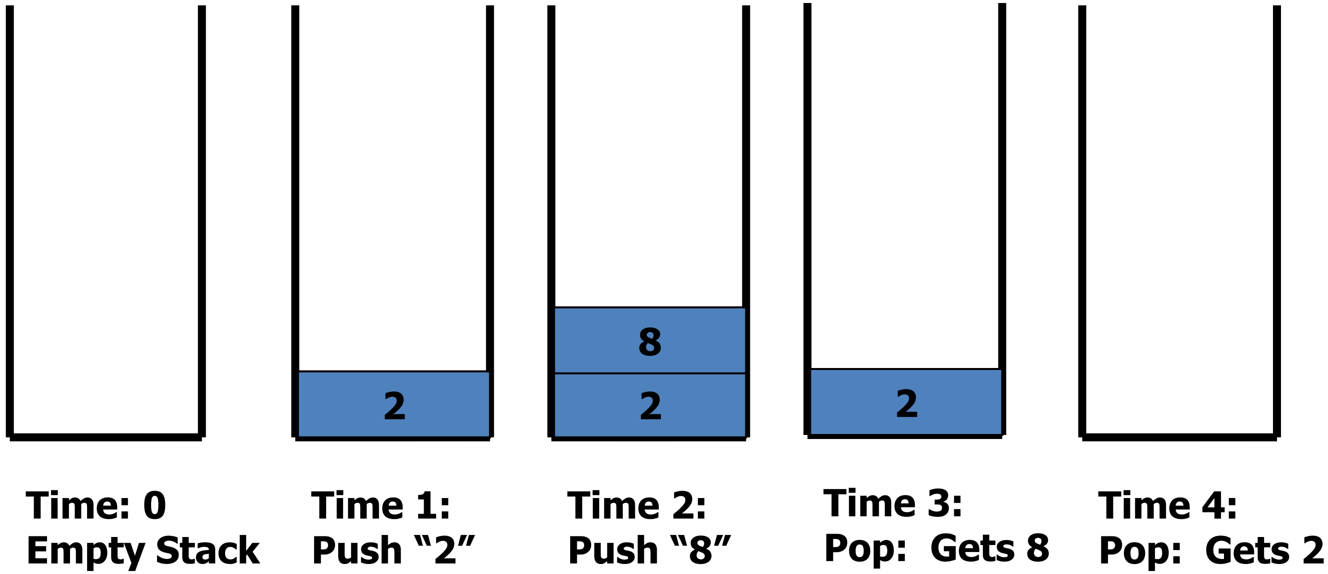
Visualizing Recursion

- To understand how recursion works, it helps to visualize what’s going on.
- To help visualize, we will use a common concept called the *Stack*.
- A stack basically operates like a container of trays in a cafeteria. It has only two operations:
  - Push: you can push something onto the stack.
  - Pop: you can pop something off the top of the stack.
- Let’s see an example stack in action.

Stacks

Stacks

- The diagram below shows a stack over time.
- We perform two pushes and two pops.



Stacks

- In computer science, a **stack** is a **last in, first out**(LIFO) abstract data type and data structure.
- A stack can have any abstract data type as an element, but is characterized by only two fundamental operations, the **push** and the **pop**.
- The push operation adds to the top of the list, hiding any items already on the stack, or initializing the stack if it is empty.

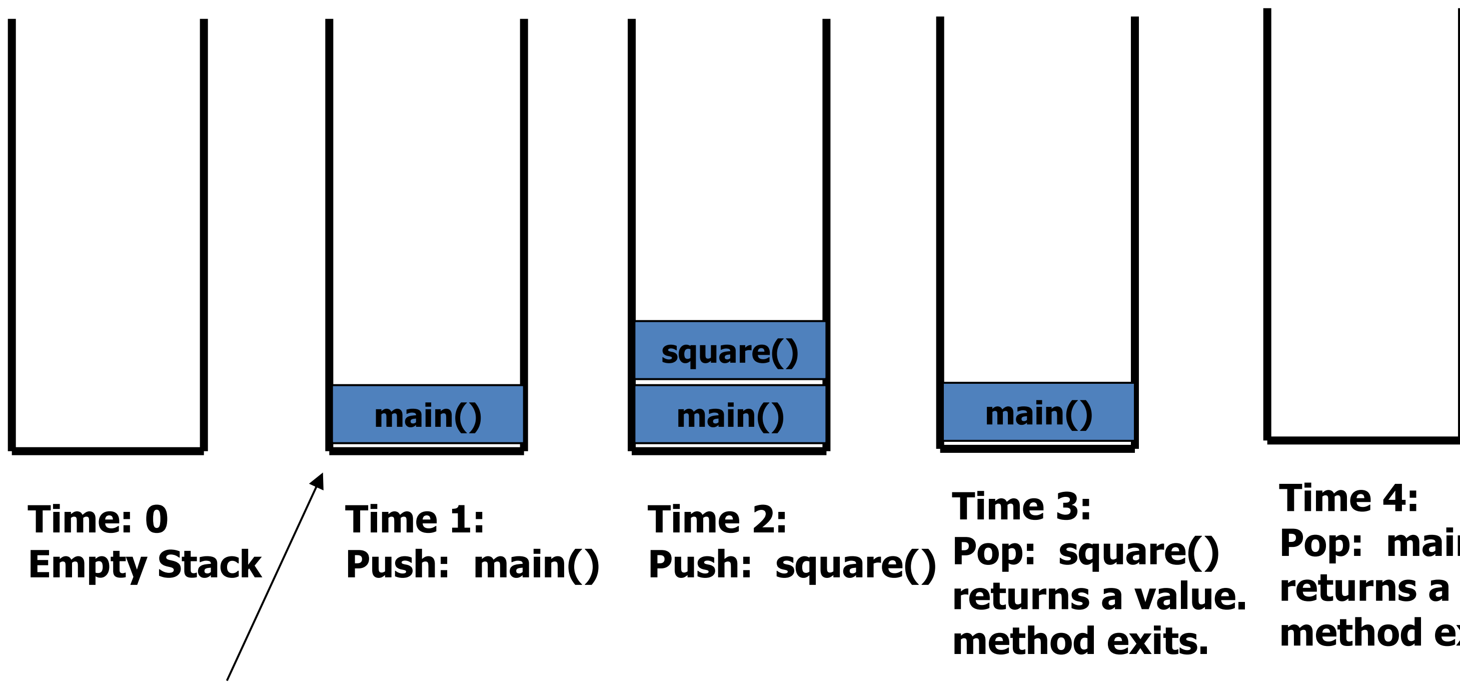
Stacks

- The nature of the pop and push operations also means that stack elements have a natural order.
- Elements are removed from the stack in the reverse order to the order of their addition: therefore, the lower elements are typically those that have been in the list the longest.

Stacks and Functions

- When you run a program, the computer creates a stack for you.
- Each time you invoke a function, the function is placed on top of the stack.
- When the function returns or exits, the function is popped off the stack.

Stacks and Functions



Time: 0
Empty Stack

Time 1:
Push: main()

Time 2:
Push: square()

Time 3:
Pop: square()
returns a value.
method exits.

Time 4:
Pop: main()
returns a value.
method exits.

This is called an activation record or stack frame.

Usually, this actually grows downward.

Stacks and Recursion

- Each time a function is called, you *push* the function on the stack.
- Each time the function returns or exits, you *pop* the function off the stack.
- If a function calls itself recursively, you just push another copy of the function onto the stack.
- We therefore have a simple way to visualize how recursion really works.

Back to the Simple Recursion Program

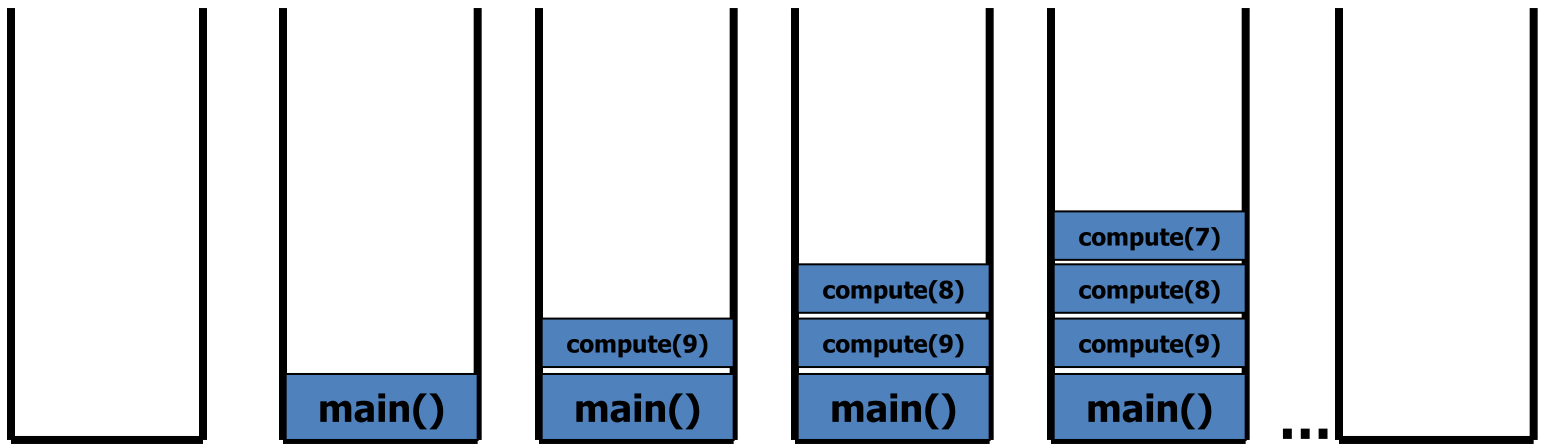
```
def compute(intInput):
    print(intInput)
    if (intInput > 2):
        compute(intInput-1)

def main():
    compute(50)

main()
```

Here’s the code again. Now, that we understand stacks, we can visualize the recursion.

Stack and Recursion in Action



Time: 0
Empty Stack

Time 1:
Push: main()

Time 2:
Push: compute(9)

Time 3:
Push: compute(8)

Time 4:
Push: compute(7)

After returning from compute(2) pop everything

```
Inside compute(9):
print (intInput);        → 9
if (intInput > 2)

     compute(intInput-1);
```

```
Inside compute(8):
print (intInput);        → 8
if (intInput > 2)
     compute(intInput-1);
```

```
Inside compute(7):
print (intInput);        → 7
if (intInput > 2)
     compute(intInput-1);
```

Defining Recursion

Terminology

```
def f(n):
  if n == 1:
   return 1
  else:
   return f(n - 1)
```

base case

recursive case

"Useful" recursive functions have:

- at least one *recursive case*
- at least one *base case*
  so that the computation terminates

Recursion

```
def f(n):
  if n == 1:
    return 1
  else:
    return f(n + 1)
```

```
Find f(5)
```

We have a base case and a recursive case. What's wrong?

Recursion

The recursive case

should call the function

on a *simpler input*,

bringing us closer and closer

to the base case.

Recursion

```
def f(n):
  if n == 0:
   return 0
  else:
   return 1 + f(n - 1)
```

```
Find f(0)
Find f(1)
Find f(2)
Find f(100)
```

Recursion

```
def f(n):
   if n == 0:
    return 0
   else:
    return n + f(n - 1)

f(3)
3 + f(2)
3 + 2 + f(1)
3 + 2 + 1 + f(0)
3 + 2 + 1 + 0
6
```

Factorial

- $4! = 4 \times 3 \times 2 \times 1 = 24$

Factorial

- Does anyone know the value of 9! ?
- 362,880
- Does anyone know the value of 10! ?
- How did you know?

Factorial

- $9! = \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
- $10! = 10 \times \quad 9!$
- $n! = n \times (n - 1)!$
- That's a recursive definition!

Factorial

```
def fact(n):
    return n * fact(n - 1)
```

fact(3)

$3 \times \text{fact}(2)$

$3 \times 2 \times \text{fact}(1)$

$3 \times 2 \times 1 \times \text{fact}(0)$

$3 \times 2 \times 1 \times 0 \times \text{fact}(-1)$

...

Factorial

- What did we do wrong?

- What is the base case for factorial?

Any Other Questions?

Announcements

- Project 1 is out
  - Due by Monday, April 18th at 8:59:59 PM
  - Do NOT procrastinate!

- Next Class: Recursion